Appendix to the article:

Petermann M, Wyler D. A pitfall in estimating the effective reproductive number Rt for COVID-19. Swiss Med Wkly. 2020;150:w20307

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Appendix 2: Suggested improvements to the analysis

The simple analysis described above only gives a first indication of the reproductive numbers around the lockdown. In order to more precisely quantify the effects of the measures on the reproductive number, this analysis should be extended to include at least the following elements.

Find change points in a systematic way

Instead of choosing the change points by a priori reasoning as above, one can infer them from the data by replacing the above A by

$$\begin{aligned} A = \{R = &(R_t)_{t=\text{Feb 24},\dots,\text{Apr 27}} : \exists(\tau_i)_{i=1,\dots,6} \text{ with } t_0 < \tau_1 < \dots < \tau_6 \text{ such that} \\ &R \text{ is piecewise linear with change points at } \tau_1, \dots, \tau_6 \text{ and} \\ &R_{\text{Feb 24}} = R_{\text{Mar 05}} = 3.30, R_{\tau_1} \geq R_{\tau_2} \geq \dots \geq R_{\tau_5}, R_T = R_{\tau_6}\}. \end{aligned}$$

Weekend effects

Weekend effects can be taken into account by substituting the above C_t by

$$C_t = \sum_{s \leq t} \sum_{i=1}^{I_s} \delta_t(s + Z_{s,i} + \zeta_{s+Z_{s,i},i}).$$

Here, $\zeta_{t,i}, i \geq 1$, are i.i.d. copies of $\zeta_{wd(t)}$, where $wd(t) \in \{\text{Mon, Tues}, \dots, \text{Sun}\}$ denotes the weekday of t. $P(\zeta_{\text{Sat}} = 2)$ is then the probability, that a test that without weekend effects would have been performed on a Saturday was actually performed on the following Monday. These probabilities could be estimated directly from the confirmed cases, or they can be fitted together with the other parameters. For the expected number of confirmed cases on day t we get then

$$C_t^{av} = E\big[\sum_{s \leq t} \sum_{i=1}^{I_s} \delta_t(s + Z_{s,i} + \zeta_{s+Z_{s,i},i})\big] = \sum_s K_{t,s} I_s,$$

with

$$K_{t,s} = \sum_r P(Z = r - s) P(\zeta_{wd(r)} = t - r).$$

Impact of quarantining on infection intensities

A natural guess for the infection intensities w_n^{qu} of those that quarantine starting the day after the onset of symptoms is

$$w_n^{qu} = \frac{w_n P(X \geq n)}{\sum_{k \geq 1} w_k P(X \geq k)}, n \geq 1,$$

where X denotes the incubation time.

The average infection intensities at day t are then

$$\gamma_t w_n + (1 - \gamma_t) w_n^{qu},$$

where $\gamma_t \in [0, 1]$ is the proportion of infectives on day t that do not quarantine although symptomatic. Thus, quarantining can be taken into account by minimizing over $A \times \mathbb{R} \times G$, where G consists of an adequate set of admissible γ-curves.