Appendix to the article:

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Appendix 1: Fitting algorithm

Our algorithm leading to the curves I^{fit} and R^{fit} of Fig. 3 can be viewed as a deterministic or non-Bayesian version of the analysis performed in (7) to estimate the effects of non-pharmaceutical interventions on COVID-19 in Europe. We give in the following the details of our algorithm.

As in the main text, we denote by $w_n, n \geq 1$, the infection intensities and by R_t the effective reproductive number on day t.

We assume that no infections were present in Switzerland before t_0 = February 24, 2020, i.e. for $t < t_0$ we set $I_t = 0$.

The period between t_0 and t_1 = March 06 is taken as the burn-in phase, where we assume that each day λ infections are imported and that these already lead to secondary infections, i.e. for $t = t_0, t_0 + 1, \ldots, t_1 - 1$, we set

$$I_t = R_t \sum_{n \geq 1} I_{t-n} w_n + \lambda.$$

After t_1, we assume that the "internal" secondary infections dominate such that imported infections can be neglected, i.e. for $t \geq t_1$ the number of new infections I_t are given by

$$I_t = R_t \sum_{n \geq 1} I_{t-n} w_n.$$

In particular, we assume throughout that the number of new infections is a deterministic function of the imported infections and of the reproductive numbers, i.e. that random fluctuations in the number of infections can be neglected.

A posteriori, these choices turn out to be consistent. According to our fit of the new infections per day, we get approximately 500 new cases for March 06. Thus, the neglected fluctuations are with a fairly large probability within twice the square root of 500, i.e. within ± 45, while the fitted value for λ is approximately 47.

We denote by Z the (random) number of days between infection and confirmation by virus test. Then the number of cases that are tested on day t and confirmed positive is given by

$$C_t = \sum_{s \leq t} \sum_{i=1}^{I_s} \delta_t(s + Z_{s,i}),$$

where $Z_{s,i}$ are independent and identically distributed (i.i.d.) copies of Z and $\delta_t(x) = 1$, if $x = t$, and 0 otherwise.

We denote the probability measure of the common probability space of all these random variables by P, and expected values with respect to P by E. For the expected number $C_t^{av} = E[C_t]$ of confirmed cases on day t we get

$$C_t^{av} = C_t^{av}(R, \lambda) = E\big[\sum_{s \leq t} \sum_{i=1}^{I_s} \delta_t(s + Z_{s,i})\big] = \sum_{s \leq t} I_s P(Z = t - s),$$

where we make explicit, that this series depends on the series of reproductive numbers, $R = (R_t)$, and on the number λ of daily imported infections during the burn-in phase.

Next, we denote by C_t^{true} the true number of cases confirmed positive on day t. For determining the best fit, we use the L^2-distance expression in the form[1]

$$\chi^2(R, \lambda) = \sum_{t \le T} \big(\sum_{s \le t} C_s^{av}(R, \lambda) - \sum_{s \le t} C_s^{true} \big)^2,$$

where the last information taken into account is that of $T =$ April 27, 2020, the last number of confirmed cases not affected by the first relaxation of measures of April 27.

Then we set

$$(R^{fit}, \lambda^{fit}) = \underset{(R,\lambda) \in A \times \mathbb{R}}{\operatorname{argmin}} \chi^2(R, \lambda),$$

where A is a suitable set of admissible R-curves, encoding a priori assumptions to reduce the number of parameters to be fitted, which is necessary, because minimization over all R-curves would lead to overfitting. The a priori assumptions made by us are as follows.

It is reasonable to assume that the R_t do not fluctuate largely from day to day, except possibly around a limited number of "change points", where a change of measures is put in force. Therefore, we have chosen A to consist of piecewise linear R-curves. As we concentrate on the days around the lockdown, we limit ourselves to a minimal number of pieces. The natural choices for the change points are $t_2 =$ March 12 (the last day of "normal life"), $t_3 =$ March 13 (the press conference announcing first drastic measures), $t_4 =$ March 17 (lockdown), $t_5 =$ March 20 (ban of gatherings > 5 people), $t_6 =$ March 27 (intermediate change point). After that, we assume that the reproductive number stays constant. (Here, the intermediate change point is introduced to test that after the "ban of gatherings > 5 people" the reproductive number stays effectively approximately constant. If this was not the case, then the choice of change points needed revision.) Moreover, the case numbers are too low to estimate the reproductive numbers in the burn-in phase. Therefore, we rely for this period on typical estimates found in the literature and set $R_t = 3.30$, for t between February 24 and March 05. Finally, as up to March 20 measures are strengthened, but never relaxed, we require that the admissible R-curves do not increase before t_5. Summarizing, we choose

$$\begin{aligned} A = \{R =&(R_t)_{t=\text{Feb }24,\ldots,\text{Apr }27} : R \text{ is piecewise linear with change} \\ &\text{points at } t_1, \ldots, t_6 \text{ and } R_{\text{Feb }24} = R_{\text{Mar }05} = 3.30, \\ &R_{t_1} \ge R_{t_2} \ge \cdots \ge R_{t_5}, R_T = R_{t_6}\}, \end{aligned}$$

[1] The reason, why we choose the L^2-distance between the cumulative counts and not between the daily counts, is that testing shows a weekly pattern due to non-corona related effects. Its impact shows less in the cumulative view. We refer to appendix B for a more direct way to take these effects into account.

i.e. we fit 6 reproductive numbers, $R_{\mathrm{Mar}\,06}, R_{\mathrm{Mar}\,12}, R_{\mathrm{Mar}\,13}, R_{\mathrm{Mar}\,17}, R_{\mathrm{Mar}\,20}$, $R_{\mathrm{Mar}\,27}$, plus λ.

We would like to emphasize that the reproductive numbers resulting from this fitting procedure are quite unstable, in particular with respect to variations of the distributions of Z and of the infection intensities w_n that are not inferred here but estimated elsewhere (3,9). Therefore, R^{fit} should at most be considered a likely picture of reality. To get more confidence in the results, a thorough sensitivity analysis would be required, confidence intervals should be calculated (see e.g. the above mentioned (7), and improvements to the analysis like the ones described in appendix 2 schould be incorporated.